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COMPARATIVE ANALYSIS OF SOME FORMULATIONS OF THE STABILITY PROBLEM

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Various formulations of the problem of stability of uniaxial tension of a band under superplastic strains are analyzed. A more general formulation (as compared to those available) is proposed; criteria of stability are obtained. Various methods of describing the behavior of materials in the superplasticity regime are considered.

Key words: *stability, uniaxial tension, superplastic strains, methods of motion description.*

The basis for the mathematical model of all physicomathematical processes is known to be the constitutive relation (CR). There are numerous constitutive relations obtained empirically for the description of superplasticity (SP) (see, e.g., [1]), but it is still a difficult task to choose a necessary CR among this set in studying a particular process. Therefore, a procedure should be developed for evaluation of CRs from the viewpoint of their applicability for describing the regime of superplastic deformation (SPD). In this paper, we analyze the available formulations and solutions of the problem of stability of the deformation process to small or finite perturbations of the current configuration of the sample [2] and their applicability for the description of the SP process.

In the present paper, we consider only the process of uniaxial tension of the sample, because of its high sensitivity to free surface perturbations. Stability of homogeneous deformation with respect to small perturbations is analyzed; hence, the approach described is inapplicable to materials that experience SPD under uniaxial tension due to inhomogeneous strains. We also avoid considering materials where SPD occurs in stable inhomogeneous regimes of uniaxial deformation (for instance, those with a "running neck"). Moreover, only those materials where the so-called structural SP is observed [1] are considered in the paper.

In SPD research, the method of motion description (Lagrangian or Eulerian) is of great importance. Klyushnikov [3] noted that there is a problem of choosing the method of motion description in studying stability of liquid flows. Thus, it seems reasonable to analyze the previously proposed approaches and the results obtained in more detail.

Analysis of Hart's Formulation of the Stability Problem. Hart was one of the first researchers who proposed to consider SP as a deformation regime stable to small perturbations of the sample geometry. Using the Lagrangian approach to motion description, Hart considered unstable regimes of homogeneous deformation and derived a criterion, which implies that the capability of materials to long tensions is retained even if there are inhomogeneities in the sample geometry [4]. Uniaxial tension of a homogeneous sample was considered under the following assumptions:

1) dependence of the "true" stress σ on the loading history and linear dependence of its small variations on the plastic strain ε and strain rate $\xi \equiv \dot{\varepsilon}$;

2) homogeneity of deformation, which implies that the stress σ at each point of the sample is equal to the ratio of the tensile force F to the cross-sectional area S at the current time: $F = \sigma S$;

3) material incompressibility: $SL = \text{const}$ (L is the sample length at the current time).

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Under these assumptions, the problem of deformation of a geometrically inhomogeneous rod in the SP regime was solved. A sample with a small part having a cross section was considered. The area of this cross section differs from the area of the remaining part by δS (the so-called "neck"). As a stress–strain state arises in the course of deformation of the inhomogeneity, the assumption 2 is not completely correct: Hart noted that this assumption induced an error of the same order of smallness as δS [4]. It should be noted that this statement is not completely correct either: Hart [4] did not say anything about the perturbation geometry, which is an important characteristic of the sample and which determines the inhomogeneity of the stress distribution.

Hart formulated the stability criterion as follows: the deformation is stable if $(\delta \dot{S}/\delta S)_F \leqslant 0$ and unstable if $(\delta \dot{S}/\delta S)_F > 0$. In other words, the growth rate of the variation of the cross-sectional area $\delta \dot{S}$ and the area perturbation δS should have different signs for the perturbation to decay; otherwise, the perturbation is enhanced. As the force F is identical at all points of the rod, we can obtain the following relations using the assumptions 1–3:

$$\delta F = \delta\sigma\, S + \sigma\, \delta S = 0, \qquad \delta\sigma = \frac{\partial\sigma}{\partial\varepsilon}\,\delta\varepsilon + \frac{\partial\sigma}{\partial\xi}\,\delta\xi,$$

$$\delta\varepsilon = \frac{\delta L}{L} = -\frac{\delta S}{S}, \qquad \delta\xi \equiv \delta\dot{\varepsilon} = -\frac{\delta\dot{S}}{S} + \frac{\dot{S}}{S}\,\frac{\delta S}{S}.$$

The third equality follows from the property of incompressibility. Combining these relations and taking into account that $\xi > 0$, we can write the stability criterion in the form

$$\Big(\frac{\delta\dot{S}}{\delta S}\Big)_F \sim \frac{1-\gamma-m}{m} \leqslant 0. \tag{1}$$

With respect to phenomenological parameters, criterion (1) can be written in the form $\gamma + m \geqslant 1$. Here $\gamma \equiv (1/\sigma)(\partial\sigma/\partial\varepsilon)$ is the strain hardening parameter and $m \equiv (\xi/\sigma)(\partial\sigma/\partial\xi)$ is the sensitivity of the material to the strain rate. In the general case, these parameters are material functions. For viscous materials, $\gamma = 0$; therefore, the stability criterion takes the form $m \geqslant 1$. Hart told that the stability criterion was the main reason for the existence of superplasticity [1].

Though Hart performed an engineering analysis (for instance, the assumption 2 is a rigorous assumption, because the strains and stresses cannot be considered as homogeneous if the cross-sectional area is not identical along the sample), his results are in good agreement with experimental data.

Following Hart, we analyze the possibility of using the solutions of the stability problem to check the adequacy of the SP constitutive relation. The papers of Il'yushin [5], Ishlinskii [6], and Keppen and Rodionov [7] considered below, which describe the formulations and solutions of the problem of stability of deformation of samples made of a viscoplastic material, have nothing to do with SP at first glance. It should be noted, however, that SPD is most often described with the use of viscoplastic models; for this reason, we analyze here the results of [5–7].

Analysis of Il'yushin's Formulation of the Stability Problem. The problem of stability of a viscoplastic flow with the use of the Lagrangian method for motion description was posed and solved by Il'yushin [5]. The motion is unstable or stable, depending on whether the perturbation is amplified or decays with time. In contrast to [4], Il'yushin [5] described the problem formulation in more detail and presented a universal method for solving this problem.

The problem of uniaxial tension of a viscoplastic solid in the form of a band of length $2l$ and width $2h$ under the action of forces $\pm 2P$ applied to the band ends and aligned in the opposite directions is solved. For convenience of comparisons of the results obtained, we consider here a uniform formulation of the stability problem, which can be easily transformed to the formulation used in [5]. The formulation includes the equation of equilibrium, the condition of incompressibility, and the CR for a linearly viscoplastic solid:

$$\hat{\nabla}\cdot\sigma = \mathbf{0}, \qquad \hat{\nabla}\cdot\boldsymbol{V} = 0,$$
$$\sigma = -pI + (\tau(\xi)/\xi)D, \qquad \tau(\xi) = K + \mu\xi. \tag{2}$$

Here $\hat{\nabla} = \hat{\boldsymbol{e}}^i\,\partial/\partial\zeta^i$ is the material operator of the gradient in the current configuration, $\hat{\boldsymbol{e}}^i$ is the vector of the local Lagrangian basis in the current configuration, ζ^i are the Lagrangian coordinates (for a two-dimensional problem, $i = 1, 2$), σ is the Cauchy stress tensor, D is the strain rate tensor, p is the hydrostatic pressure, I is the unit tensor of the second rank, $\tau = \sqrt{2\sigma' : \sigma'}$ is the intensity of shear stresses (the prime marks the deviatoric part of σ),

$\tau(\xi) = K + \mu\xi$ for a viscoplastic medium, K is the plasticity coefficient, μ is the viscosity, $\xi = \sqrt{D : D/2}$ is the intensity of the shear strain rate, and $\boldsymbol{V}$ is the vector of motion velocity. All fields of the variables are functions of the Lagrangian coordinates.

The boundary conditions have the following form:

— on the side surface $[x^2 = \pm h(t)]$,

$$\hat{\boldsymbol{n}} \cdot \sigma = \boldsymbol{0} \tag{3}$$

($\hat{\boldsymbol{n}}$ is the outward normal to the sample surface in the current configuration);

— on the end faces of the sample $[x^1 = \pm l(t)]$,

$$\sigma_{11} = P(t)/h, \qquad \sigma_{12} = 0. \tag{4}$$

In Eqs. (3), (4), x^1 and x^2 are the spatial coordinates that are functions of the Lagrangian coordinates ζ^1 and ζ^2, respectively.

Using the method of solving this problem, which was described in [5], we can obtain the stability criterion. Equations (2) and the boundary conditions (3), (4) are written in terms of perturbations of the stress function and stream function, which are expanded into the Fourier series near the state of the band under uniaxial tension; as a result, Eqs. (2) reduce to a system of linear differential equations with respect to unknown perturbations. The solution of these equations satisfy the boundary conditions (3) and (4), including the conditions along the surfaces where the total perturbation $\bar{\delta}$ is presented in the form of the Fourier series $\bar{\delta} = \sum_{n=1}^{\infty} \delta_n \cos(\pi n x^1/l)$, and allow us to find the velocity components. Instability of motion depends on the sign of the velocity components $V_2(h)$ of motion arising on the perturbed surface: the sign may coincide with or be the opposite to the sign of the perturbation $\bar{\delta}$. Thus, the study of flow stability reduces to studying stability of individual components $\delta_n \cos(\pi n x^1/l)$ of the perturbation $\bar{\delta}$: the qth component is stable (i.e., increases), and the motion with respect to this component is unstable if the inequality $V_2(h)\cos(\pi q x^1/l)/\delta_q > 0$ holds. If the qth component is the only stable component, it starts to prevail over all other components with time; finally, the shape of the boundary transforms to a cosinusoidal curve forming q waves along the band. Band deformation is unstable if the only component of the perturbation is stable. Solving the last inequality, we finally obtain the criterion of stability of the component and instability of motion in the form

$$2n < 2q\,\frac{h}{l}\sqrt{\frac{\chi}{\chi + \xi_0}} < 2n + 1, \qquad n, q \in \mathbb{Z}, \quad \chi = \frac{K}{\mu}, \tag{5}$$

where ξ_0 is the strain rate in the case of uniaxial tension of the sample and $\mathbb{Z}$ is a set of integers.

Analysis of Ishlinskii's Formulation of the Stability Problem. Ishlinskii [6] presented the solution of the stability problem for a viscoplastic flow of a band with the use of the Eulerian method of motion description. By analogy with the above-performed analysis of Il'yushin's results, we use the results obtained by Ishlinskii in studying the stability of the flow of a linearly viscous sample to verify the adequacy of the SP constitutive relation. For convenience, the mathematical formulation of the problem is written in the same form as in [5] [Eqs. (2) and boundary conditions (3) and (4)]. The difference of this formulation is that all fields are functions of the Eulerian coordinates x^i $(i = \overline{1, 2}\,)$. We study the flow of a rectangular band whose edges perpendicular to the axis x^1 have a straight-line form at the initial time. The edges parallel to this axis are subjected to a small "perturbation" described by the equations

$$x^2 = h + \delta\cos(ax^1), \qquad x^2 = -h - \delta\cos(ax^1),$$

where h is the half-width of the unperturbed band and δ is the perturbation amplitude, which is small as compared with h. The perturbation of the boundary has a sinusoidal form symmetric about the axis x^1. The band length is assumed to accommodate an integer number of perturbation half-waves $a = \pi q/l$ (q is an integer number and l is the sample length). If the perturbation of the band edges parallel to the axis x^1 has a more complicated character, it can be presented as a sum of simple sinusoidal perturbations by using the Fourier series.

The perturbation has a tendency to increase if the velocity component V_2 has the same sign as the perturbation $\eta = \delta\cos(ax^1)$, i.e., if $V_2/\eta > 0$. Solving this inequality, we can write the stability criterion in the form

$$2n < 2q\,\frac{h}{l}\sqrt{\frac{K}{K+\mu\xi_0}} < 2n+1, \qquad n, q \in \mathbb{Z}. \tag{6}$$

Criterion (6) coincides with Il'yushin's stability criterion (5), though it was obtained by the Eulerian approach. Thus, by an example of papers [5, 6], we demonstrated the equivalence of the Lagrangian and Eulerian approaches to solving the problem of stability of uniaxial tension of a band made of a linearly viscoplastic material.

Analysis of Keppen and Rodionov's Formulation of the Stability Problem. The logical continuation of Il'yushin's research was the paper of Keppen and Rodionov [7], where they considered tension and compression of a band made of a nonlinearly viscoplastic material. In contrast to Il'yushin's paper [5], which implied a linear dependence between the strain rate ξ and the shear stress intensity τ, Keppen and Rodionov [7] used a more general relation

$$\tau(\xi) = K + \mu\xi_0 g(\xi/\xi_0), \tag{7}$$

where $g(\xi/\xi_0) > 0$ is a certain function of ξ.

Similar to [5], the stability criterion is sought for a problem whose formulation can be reduced to the form (3)–(5) with allowance for (7): band tension is unstable if

$$2n < 2q\,\frac{h}{l}\sqrt{1-\frac{\tau'(\xi_0)\xi_0}{\tau(\xi_0)}} < 2n+1, \qquad n, q \in \mathbb{Z}. \tag{8}$$

We analyzed the results of theoretical papers where the SP process is considered as a stable process of tension in the case of imperfections on the side surface. Hart's paper [4] gave an impetus to the development of this research direction, but it had some drawbacks (one-dimensional analysis of neck development was performed, and rather rigorous restrictions were imposed on the process considered). Il'yushin [5] described details of a universal method of solving problems of stability of tension of a viscoplastic sample. With the help of this method, solutions of a two-dimensional problem of neck development for a linearly viscoplastic sample were obtained in [5, 6] on the basis of different approaches (Eulerian and Lagrangian). Keppen and Rodionov [7] extended Il'yushin's problem to the case of a nonlinearly viscous material and solved the resultant problem with the Lagrangian approach.

In the present paper, we propose a more general formulation of problems of band tension and compression in the case of a nonlinearly viscous rheology of the medium. Such a formulation allows us to use various types of perturbations, and all formulations described above follow from the formulation proposed here. Moreover, all problems of stability of uniaxial tension considered above are geometrically linear. A homogeneous process of evolution of geometric imperfections of the sample are considered in these problems. As was shown by Wray [8], however, the real process is more complicated (growth and interaction of "necks"); hence, the perturbations of the configuration cannot be neglected. A geometrically nonlinear formulation of the problem of stability of uniaxial tension of a sample made of a nonlinearly viscous material is proposed in the present paper.

Problem of Stability of Uniaxial Tension of a Flat Band Made of a Nonlinearly Viscous Material. We consider a geometrically nonlinear problem of stability of uniaxial tension of a flat rectangular sample with a constant relative velocity of its ends to small normal perturbations of the configurations at a fixed time (that is why the time is not included into the list of independent variables). The problem is solved for the case of a plane strain state. The mathematical formulation of this problem includes the equations of motion, the constitutive relations of the general form, and the incompressibility conditions:

$$\hat{\nabla}\cdot\sigma = \mathbf{0}, \qquad \sigma = -pI + (\tau(\xi)/\xi)D, \qquad \hat{\nabla}\cdot\boldsymbol{V} = 0. \tag{9}$$

Here $\tau(\xi)$ is an arbitrary nonlinear function of intensity of the strain rate tensor ξ (the formulations for a linearly viscous sample given above can be derived from this formulation with $\tau(\xi) = K + \mu\xi$).

The boundary conditions have the following form:

— on the side surface,

$$\hat{\boldsymbol{n}}\cdot\sigma = \mathbf{0} \tag{10}$$

($\hat{\boldsymbol{n}}$ is the outward normal to the side surface of the sample in the current configuration);

— on the end faces of the sample,

$$V_1\Big|_{x^1=0} = 0, \quad V_1\Big|_{x^1=l} = V_0 > 0, \quad \sigma_{12}\Big|_{x^1=0} = \sigma_{12}\Big|_{x^1=l} = 0. \tag{11}$$

Let us use various approaches to motion description and compare the resultant stability criteria.

Let us first consider the Lagrangian approach where all fields of variables are functions of the Lagrangian coordinates. Let the fields of velocities, pressure, and Hamiltonian operator (variation of the configuration) experience small perturbations at a certain time: $\boldsymbol{V}_{\rm p} = \boldsymbol{V} + \delta\boldsymbol{V}$, $p_{\rm p} = p + \delta p$, and $\hat{\nabla}_{\rm p} = \hat{\nabla} + \delta\,\hat{\nabla}$. Then the formulation of the problem for perturbations has the form

$$\delta\hat{\nabla}\cdot\sigma + \hat{\nabla}\cdot\delta\sigma = \boldsymbol{0}, \qquad \delta\hat{\nabla}\cdot\boldsymbol{V} + \hat{\nabla}\cdot\delta\boldsymbol{V} = 0,$$
$$\delta\sigma = -\delta pI + (\tau'/\xi - \tau/\xi^2)\delta\xi\, D + (\tau/\xi)\,\delta D; \tag{12}$$

$$\delta\hat{\boldsymbol{n}}\cdot\sigma + \hat{\boldsymbol{n}}\cdot\delta\sigma = \boldsymbol{0}. \tag{13}$$

Here $\delta D = (\delta\hat{\nabla}\boldsymbol{V} + \hat{\nabla}\delta\boldsymbol{V} + \delta\boldsymbol{V}\,\hat{\nabla} + \boldsymbol{V}\delta\hat{\nabla})/2$ and $\delta\xi = ((D_{11}-D_{22})(\delta D_{11}-\delta D_{22}) + (D_{12}-D_{21})(\delta D_{12}-\delta D_{21}))/\xi$. The difference between the geometrically nonlinear analysis presented here and the analysis of the previously available formulations is the perturbation of the configuration (perturbation of the nabla-operator). From the kinematic relation $\dot{\hat{\nabla}} = -L^{\rm t}\cdot\hat{\nabla}$, we find the perturbation $\delta\hat{\nabla}$ using, instead of the differential, the variation $\delta\hat{\nabla} = -\delta u\, L^{\rm t}\cdot\hat{\nabla}$; δu is the variation of the process parameter and $L = \boldsymbol{V}\hat{\nabla}$ is the transposed velocity gradient. Using the relations $\dot{F} = L\cdot F$ and $\hat{\nabla} = \nabla\cdot F^{-1}$ (F is the transposed place gradient) or the relations written in variations

$$\delta F = \delta u\, L\cdot F \quad\Rightarrow\quad \delta u\, L = \delta F\cdot F^{-1} = (\delta\hat{\boldsymbol{r}}\,\nabla)\cdot F^{-1} = \delta\hat{\boldsymbol{r}}\,\hat{\nabla},$$

we can find the variation of the nabla-operator $\delta\hat{\nabla} = -(\hat{\nabla}\,\delta\hat{\boldsymbol{r}})\cdot\hat{\nabla}$. Then $\delta\hat{\nabla}\cdot\sigma = -((\hat{\nabla}\,\delta\hat{\boldsymbol{r}})\cdot\hat{\nabla})\cdot\sigma \equiv \boldsymbol{0}$ (because the components of σ are homogeneous in the ground state). Substituting this expression into Eqs. (12) and (13) and taking into account the first and third equations in (9), we obtain the final form of the problem formulation for a nonlinearly viscous medium:

$$\hat{\nabla}\cdot\delta\sigma = \boldsymbol{0}, \qquad \delta\hat{\nabla}\cdot\boldsymbol{V} + \hat{\nabla}\cdot\delta\boldsymbol{V} = 0,$$
$$\delta\sigma = -\delta p\, I + (\tau'/\xi - \tau/\xi^2)\delta\xi\, D + (\tau/\xi)\,\delta D. \tag{14}$$

The stability criterion is determined in accordance with the approach proposed in [5]. We use perturbations in the form

$$\delta r_1 = \varphi_1(ax^2)\sin(ax^1)\,{\rm e}^{\lambda t}, \qquad \delta r_2 = \varphi_2(ax^2)\cos(ax^1)\,{\rm e}^{\lambda t},$$

where $\hat{\boldsymbol{r}}(r_1, r_2)$ is the radius vector in the current configuration, $a = \pi q/l$ (q is an integer number and l is the sample length), x^1 and x^2 are the spatial coordinates, which are functions of the Lagrangian coordinates ζ^1 and ζ^2, respectively, and λ is the decay decrement. Then, we obtain the stability criterion in the form

$$\lambda = \xi_0\Big(-1 + \frac{2\sin(2b\sqrt{1-m}\,)}{m\sin(2b\sqrt{1-m}\,) + \sqrt{m(1-m)}\,\sinh\,(2b\sqrt{m}\,)}\Big) > 0, \tag{15}$$

where ξ_0 is the strain rate in the case of uniaxial tension of the sample, $b = ah$ (h is the sample half-width), and $m \equiv \partial\ln\tau/\partial\ln\xi = \xi\tau'/\tau$.

Let us now consider the Eulerian approach.By analogy with the material derivative for the function specified in the Eulerian coordinates ($\boldsymbol{0} = d\nabla/dt = \partial\nabla/\partial t + (\boldsymbol{V}\cdot\nabla)\nabla$), we obtain a variation of the Hamiltonian operator in the form $\delta\nabla = -(\delta\boldsymbol{r}\cdot\nabla)\nabla$, where the right side characterizes the changes caused by the relative motion of the material and spatial points coinciding at the moment. In this case, the problem formulation coincides with (13) and (14), and the stability criterion coincides with (15). Thus, two approaches to solving the problem of stability of tension of a nonlinearly viscous sample in the SP regime are equivalent.

Comparison of Criteria. Relation (15) is the stability criterion for the sample in a geometrically nonlinear case. If we consider a geometrically linear formulation (in this case, there is no variation of the nabla-operator), the stability criterion is a particular case of (15) and has the form

$$\lambda = \frac{2\sin(2b\sqrt{1-m}\,)}{m\sin(2b\sqrt{1-m}\,) + \sqrt{m(1-m)}\,\sinh\,(2b\sqrt{m}\,)} > 0.$$

Solving this inequality and introducing the notation $b = ah$, $a = \pi q/l$, and $m \equiv \partial \ln \tau / \partial \ln \xi = \xi \tau'/\tau$, we obtain the stability criterion in the form

$$2n < 2q \frac{h}{l} \sqrt{1 - \frac{\tau'}{\tau} \xi} < 2n + 1, \qquad n, q \in \mathbb{Z}, \tag{16}$$

which coincides with the criterion proposed by Keppen and Rodionov (8). Assuming that $\tau(\xi) = K + \mu\xi$ in Eq. (16), we obtain the criteria derived by Il'yushin (5) and Ishlinskii (6).

To check the geometrically nonlinear criterion (15), we compare it with Hart's criterion (1) with $a \to 0$ [the parameter $a = \pi q/l$ characterizes the number of half-waves accommodated on the sample length (in Hart's paper, $q = 1$), and its tending to zero corresponds to the process of homogeneous deformation considered]. Using L'Hospital rule and reducing the like terms, we obtain

$$\lambda = \xi_0 (1 - m)/m,$$

i.e., stability is observed for $\lambda < 0$, which is equivalent to the inequalities $m < 0$ and $m > 1$. Thus, in the limiting case with $a \to 0$, the result obtained agrees with Hart's analysis [4].

Conclusions. In this paper, we analyzed various formulations where the process of superplastic flow is considered as a stable process of tension of a cylindrical sample with imperfections of the side surface. Hart's paper [4] gave an impetus to this research direction, but its drawback was a one-dimensional analysis of development of the "neck." A universal method of solving stability problems was proposed in the papers of Il'yushin [5] and Ishlinskii [6], though they seem to have nothing to do with SP at first glance. It is proposed to use this method to check SP constitutive relations at which the existence of stable regimes is possible. We compared the criteria derived by Il'yushin and Ishlinskii and demonstrated their equivalence.

All available formulations of the problem of stability of uniaxial tension are geometrically linear, but the real mechanism of growth and interaction of the "necks" can be much more complicated; hence, perturbations of the configuration (geometrical nonlinearity) cannot be neglected.

In the paper, we proposed a formulation of the problem of stability of band tension in the case of a nonlinearly viscous rheology of the medium, performed a geometrically nonlinear analysis, obtained stability criteria with the use of the Eulerian and Lagrangian approaches, and demonstrated their equivalence for the problem considered. All formulations and criteria considered above are shown to follow from the formulation proposed here and criterion (15). This formulation allows various types of perturbations to be used.

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